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MODAL PARADOX

Modal Paradox

Abstract

In this paper we are going to deal with a paradox related to metaphysical possibilities – possibilities ruled out by modal metaphysical principles. For our purposes here, the relevant modal principle that we are concerned about is linked to essentiality of the material composition of an artefact. It will be presented here some of the many different formulations for the modal paradox available in literature. There are so many different version of the paradox. Actually, each philosopher who has written about the subject matter usually offer his own version of the paradox. As soon as the paradox's version have been presented, it will be advanced some solutions to the modal paradox available in the literature. Beginning with some more general solutions which go against either essentialist theses or modal discourse itself. And then, presenting some of the more important solutions properly to the modal paradox.

Keywords: modal paradox; vagueness; modal logic; essentialism; possibility.

Resumo

Neste artigo apresento algumas versões do chamado paradoxo modal e algumas soluções disponíveis na literatura. Há inúmeras versões do paradoxo modal disponíveis.

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Aqui vamos analisar mais de perto adaptação de quatro delas: a versão apresentada por Nathan Salmon, a versão de Roderick Chisholm, além de duas versões apresentadas por Sarah Leslie. O paradoxo modal surge de duas intuições modais acerca da essência de artefactos materiais: i) uma modificação parcial na composição material de um artefacto material é possível e, ii) uma modificação total não é possível. Essas intuições modais são a base para todas as versões que serão apresentadas aqui, a excluir apenas a versão de Chisholm. A versão de Leslie é apresentada com o intuito de mostrar que talvez o fenômeno semântico da vagueza não desempenhe papel fundamental para o surgimento de paradoxos modais. Após apresentação das versões do paradoxo, apresento algumas das soluções mais importantes disponíveis na literatura.
Palavras-chave: paradoxo modal; vagueza; lógica modal; essencialismo; possibilidade.

Introduction

Perhaps you have once asked yourself how things might have been if they had been different. You have probably already found yourself thinking about how things might have been if you had been a billionaire (you would not be reading this paper) or if you had got back home earlier last night (probably something bad could not have happened). We frequently have such thoughts. We think about ourselves in different situations where something that is actually impossible could be possible. All these kinds of thoughts are related to modalities. We can think a little bit more carefully about the subject using an example: the way things actually are, it is impossible to travel from Lisbon to Rio in less than two hours, but if things had been different (if we had more advanced aviation technology, for instance), then perhaps I would be able to take that trip. The kind of modalities involved in all these cases may help us to clarify what we mean when we are talking about possibilities, although it has not been taken seriously by metaphysicists who are interested in a more fundamental and general kind of possibility; the alethic possibility which is usually linked to either logical possibility or metaphysical possibility.

One can ask if oneself could have had different parents or not. If the earth could have been bigger than it actually is. If the water could not have been H_2O. For essentialists, there are two ways to have (exemplify, instantiate, etc.) proprieties and have parts (be composed of): the essential and the accidental. An anti-essentialist may deny this distinction. However, even if one does not subscribe to any kind of essentialism, it is not obvious that he will be in a better position with respect to some version of the paradox (see Chisholm (1967)).

Quine (1976)[1] in a brief consideration of the paradox said that «you can change anything to anything by easy stages through some connecting series of possible worlds»[2].

In this paper we are going to deal with a paradox related to metaphysical possibilities – possibilities ruled out by modal metaphysical principles. For our purposes here, the relevant modal principle that we are concerned about is linked to essentiality of the material composition of an artefact. The paradoxes often play a central role in philosophy. Many philosophers have spent a lot of time trying to sort out paradoxes since the beginning of the history of philosophy. Modal paradoxes, specifically, are a family of paradoxes which are related to metaphysical modality and metaphysical modality is the area of metaphysics concerned with the ways things could have been in addition to the way they actually are.

The Paradox's Versions

In this section, I am going to present some of the many different formulations for the modal paradox available in literature. Although the main aim of this section is not critically discuss the possible solutions to the paradox, I will try to provide to the reader the necessary background to be able to follow the solutions that have been advanced in the literature. There are so many different version of the paradox. Actually, each philosopher who has written about the subject matter usually offer his (her) own version of the paradox and some of them are really small difference in style with no philosophical substance, however some others may be more substantive and even eventually more plausible. In this case, I think, may be relevant for us to have a look at the paradox's versions before to criticise or even try give a solution to them. For pragmatic reasons other than historical one, the first version to the paradox that I am going to present has been advanced by Nathan Salmon in several different places[3]. The

[1] Quine, W. V., «Worlds Away», *The Journal of Philosophy* (1976), 859-863.

[2] Quine said this in the context of a comparison between cross-time identification and cross-world identification considering that feature of cross-world identification as devastating for both modal identification and, as a consequence, modal discourse itself.

[3] Salmon, Nathan, «Modal Paradox: Parts and Counterparts, Points and Counterpoints», *Midwest Sudies in Philosophy: Studies in Essentialism* (1986), 75-120; idem, «The Logic of What Might Have Been» *Philosophical Review* (1989), 3-34; idem, *Reference and Essence*, Prometheus Books, Amherst 1981; idem, «How Not to Derive Essentialism from the Theory of Reference», *The Journal of Philosophy* (1979), 703-725.

second one has been presented by Rodrick Chisholm[4]. And, the third and last, the Sarah Leslie's version[5] which is really similar to the Hugh Chandler's[6] (1976).

Something about the essentialist framework

Some of the versions of the paradox that we are going to analyse here are explicitly related to essentialism regarding material composition of an artefact. More generally, they are associated with an essentialist thesis about the origin of an artefact (a table, for instance) which is commonly endorsed by many essentialist philosophers since its famous vindication by Kripke:

> [C]ould *this table* have been made from a completely *different* block of wood? […] [W]e can imagine making a table out of another block of wood, identical in appearance with this one, […] it seems to me that this is *not* to imagine *this* table as made of [different] wood, but rather it is to imagine another table, *resembling* this one in all external details, made of another block of wood[7].

Many philosophers who accept this essentialist claim frequently acknowledge the pertinence of the paradox, even though it is not clear whether Kripke by himself is convinced or not of its relevance. Anyway, my main aim here is to explore the conceptual framework connected to modal paradox and modality itself.

Modal paradox arises from the following two essentialist intuitions:

[E_1] The original matter of an artefact is essential to it.
[T] A slight change in the original matter of an artefact is tolerable.

Modal paradox that shows up from E and T has been taken seriously by philosophers such as Salmon (1979) (1981) (1986), Chandler (1976), Forbes (1984) (1986), Chisholm (1967) (1973) and others. Essentialist intuitions E_1 and T are supposed to be equivalent to the following two principles may be clearer:

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[4] Chisholm, Roderick, «Identity Through Possible Worlds: Some Questions», *Noûs* (1967), 1-8; idem, «Parts as Essential to Their Wholes», *The Review of Metaphysics* (1973), 581-603.

[5] Leslie, Sarah, «Essence, Plenitude and Paradox», *Philosophical Perpectives: Metaphysics* (2011), 277-296.

[6] Chandler, Hugh, «Plantinga and the Contingently Possible», *Analysis* (1976), 106-109.

[7] Kripke, Saul, *Naming and Necessity*. Blackwell, Oxford 1980, p. 114.

[E_2] A complete (sufficiently great) change in the original matter of an artefact is not possible.
[M_T] A slight change in the original matter of an artefact is possible.

E_1 and E_2 seem to be equivalent. E_1 says that for any object *O* made of *m* and any possible world w, *O* exists in w if only if *O* is made of *m* in w. Even though it may suggest that all matter is essential, it is not entailed by E_1 (in a weak reading). E_2 says something quite similar; there is no possible world where *O* was made of *n*, matter completely different of *m*. Adopting the weaker reading to E_1, it seems to be equivalent to E_2. So, you can feel free to chose which seems textually more plausible. M_T (modal tolerance) just says that it is possible to *O* to be made of a slightly different matter. So, there is a possible world where *O* was made of a slightly different matter. Although T (tolerance) does not make any explicit mention to modality, it may (in this context) be understood as saying that it is possible for *O* to be made of slightly different matter; exactly the same what is meant by M_T. Thus, this pair of principles will be interchangeably used throughout this paper, since *prima facie* they do not differ substantially in meaning. In the following, E is going to be used to refer to both E_1 and E_2 and T analogously is going to be a label to T and M_T.[8]

Salmon's case

There are several different formulations for the paradox raised from those modal principles. In the following, I will try to explain some of them as clearly and intuitively as I can. As it was mentioned above, the first version of the paradox has been advanced by Nathan Salmon in many different places to hold many different theses. However, in what follows, I will reproduce the paradox inspired mainly by what can be found in Salmon (1989).

Suppose we are talking about a table, we can call it 'Woody', made of a specific hunk of matter *m*. Woody is part of the furniture of the actual world. Now we can ask ourselves: could Woody have been made of a different matter *n*? For an essentialist, it depends. As we have seen in the previous section, essentialist consider the original matter essential to the artefact made of it, but they accept a slight change in that matter (this is exactly what E and T claim). So, for them the answer depends if *n* has sufficient overlap with the original matter *m*. If so, it is possible for Woody to be made of matter *n*. If there is no suffi-

[8] If the difference between T and M_T become to be relevant, the reader will be made aware of it.

cient overlap with *m*, then it is not possible for Woody to be made of *n*. One may contend that 'sufficient overlap' is a vague phrase which means that 'x has sufficient overlap with y' is a relational vague predicate. And it is arguable that vagueness plays a fundamental role here and the modal paradox is just an instance of the more familiar Sorites' paradox. If I am right, that is not the case and I will pursue that argument later. For now, we need not settle it here and we can consider the modal paradox just as a vagueness paradox with no risk of misunderstanding.

Once we have accepted that Woody could have been made of a slightly different matter with sufficient overlap with respect to the matter from which it was actually made, we should say (appealing to Kripke-style semantics) that there is a possible world w_1, different from the actual world w_0, where Woody was made of *n* – matter with sufficient overlap with *m*. Or, equivalently, it is possible to Woody be made of *n*. Moreover, we can ask ourselves if Woody made of matter *n* could be made of a different matter. And the answer here must be the same as the one previously given: Yes, since the new matter has sufficient overlap with *n*. So, there is a possible world w_2 where Woody was made of *o* – a matter with sufficient overlap with *n* – and w_2 is possible with respect to w_1. Or, equivalently, it is possibly possible to Woody to be made of *o.* By applying exactly the same method we will arrive at a possible world w_n where the table was made of a matter with sufficient overlap with respect to matter used to make the table in w_{n-1}, but it is completely different from the matter from which Woody was actually made. Although w_n is a possible world with respect to w_{n-1} – there was sufficient overlap between the matter by which the table was made in w_n with respect to w_{n-1} and the same happened in all the steps of reasoning –, w_n is not possible with respect to the actual world[9]. Just because there is no sufficient overlap between the matter Woody was made from (*m*) and the matter the table from w_n was made from. In other words, there is a different table rather than Woody in w_n. We can call it 'Middy'. Middy is not a possible way for Woody to be, although there are some intermediate possible ways for Woody to be with respect to which Middy is a possible way for them to be. From the way things actually are it is impossible for Woody to be Middy. Nevertheless, if things had been different (if Woody had been made of a different matter with sufficient overlap with which Middy was made), then Woody might have been Middy.

[9] For any pair of possible worlds immediately next each other w_{n-1} and w_n in the chain, the matter of the table from the last one has sufficient overlap with the matter from the former.

Chisholm's paradox

One of the first formulations of the paradox was given by Chisholm (1967) and in his formulation the paradox is not conceived in essentialist terms. In the way Chisholm presented the paradox, no essentialist principle is needed and it leads us to bizarre conclusions: first, possibly Adam and Noah are the same person. Second, possibly anyone is identical to anyone else. This is precisely the Quine's claim «you can change anything to anything by easy stages through some connecting series of possible worlds»[10]. Obviously, both of these conclusions are intolerable. Chisholm's version of the paradox may be presented as follows: let us think about two possible worlds w_0, the actual world, and w_1, an alternative possible world. Adam actually died when he was 930 years old. Could he have lived for 931 years in w_1? Or equivalently, Adam lived for 930 years in w_0, could he have lived for 931 years in w_1? If we have no essentialist principle to rule out of the question and the properties both to live 930 in w_0 and to live 931 in w_1 are not incompatible properties, then we should say that Adam holds both of them. Now we can make a slightly change in the case considering Noah in addition to Adam. Noah lived for 950 years in w_0 and we can suppose that he lived for 949 years in w_1. Once again, these proprieties are not incompatible between each other, so all of the steps of our reasoning are allowed. We can move now to a possible world w_2 where Adam lived for 932 and Noah lived for 948. Applying the same reasoning repeatedly we will arrive at a possible world where Adam lived for 950 years and Noah lived for 930 years. We can keep making little alterations in Adam and Noah, for example, change one letter of their names in each possible world ('Ndam' and 'Aoah') to reach a possible world where their names were changed. We can continue changing their properties to arrive at a far away possible world w_n where Noah has exactly the same proprieties that Adam has in w_0 and vice-versa. But $Noah_{wn}$ can be traced back to $Adam_{w0}$ and $Adam_{wn}$ can be traced back to $Noah_{w0}$. The questions now are: 1) Is $Noah_{wn}$ identical to $Adam_{w0}$ and $Adam_{wn}$ identical to $Noah_{w0}$? 2) Is w_0 identical to w_n? These questions are really puzzling and they will remain unanswered for now, but more important for us here is to argue that to deny the essentialist thesis will not prevent us from a version of the modal paradox for very long. It is also worth noting that w_0 and w_n are qualitatively indiscernible. Despite having distinctly modal histories, they have exactly the same proprieties and nothing else can be used to distinguish

[10] Quine, W. V., «Worlds Away» *The Journal of Philosophy* (1976), 859-863.

them. If we put them in front of God's eyes, then He would not be able to see any difference that justifies their simultaneously existence and He would think about himself as one who has done meaningless work.

Sorites' modal paradox and Leslie's version

As it was briefly mentioned above, one may argue that the modal paradox is not a special paradox, instead it is in fact an instance of the more familiar Sorites' paradox or vagueness paradox. If that is the case, then the paradox arises from some vague term invocated. In the first version of the paradox we have regarded in this paper, it is quite clear that there is a vague phrase and one may argue the vague phrase plays a fundamental role to paradox emergence. The vague phrase in the first version of the paradox is 'sufficient overlap'. If we take any portion of matter *m,* we cannot know which exactly is the part of matter with sufficient overlap to it, just because we do not know how much the matter can be changed keeping sufficiency overlapping. It will always be possible to make a new slight change in the matter and we will not be able to know whether there is sufficient overlap or not. If this is correct, then 'sufficient overlap' is a vague phrase similar to the words 'bald' or 'tall'. If we say about some x that he is tall, then we have to say the same about y, one centimetre shorter than x, that he is tall. Of course, we have to say about z, one centimetre shorter than y, that he is also tall. This kind of reasoning will lead us to say about someone 1,2 meter tall that he is tall, but, of course, this is not true. One who intend to avoid to modal paradox can argue that the modal paradox arises exactly the same way from the vague phrase 'sufficient overlap'. Therefore, the modal paradox is just an instance of Sorites' paradoxes and any good solution for them is simultaneously a good solution to the modal paradox. So, we should not give it special attention, instead we should merely take it as an ordinary case of vagueness.

We might partially agree with the argument just presented accepting that the phrase 'sufficient overlap' is vague. And disagree with the conclusion the modal paradox is just an ordinary instance of vagueness paradox and it does not deserve any special attention. In the following I will give you a very similar version of the paradox which will not appeal to any vague phrase. In fact, it is very easy to show that modal paradox is not a mere case of Sorites' paradox. All we need to do is to state a version of the paradox with no vague phrases and, if we are successful, then we will have showed that modal paradox has special features despite its resemblances to regular Sorites' paradoxes.

One simple variation of the modal paradox has been presented by Leslie. She thinks (and I agree) that her version of the paradox makes clear that vagueness phenomena does not play a special role to the modal paradox emergence. Leslie's formulating just presents the paradox with a stipulated essence, something as substituting 'sufficient overlap' for '98% overlap' in our original version of modal paradox.

> One might suspect that this paradox is driven by vagueness – that it is somehow no more than a sorites paradox in disguise, rather than being a sui generis paradox about essence. However, this is not so, as is made clear by another version of the paradox […] whose essence is stipulated in a precise way.[11]

In the non-vague version of the paradox we have to run an adaptation in the modal principle that we have considered to catch the idea present in the phrase '98% overlap'. Thus, we should keep E and change T for T_{F1} what may be read as 'fixed tolerance' and it is defined as follows:

> [E] The original matter of an artefact is essential to it.
> [T_{F1}] A change no bigger than 2% in the original matter of an artefact is tolerable.

Following T_F Woody might be made of a different matter overlapping at least 98% the matter from which it was actually made. So, we should say that there is a possible world w_1, different from the actual world w_0, where Woody was made of *n* – matter bigger than 98% overlapping with *m*. Woody made of matter *n* could be made of a different matter? Yes, since the new matter retains at least 98% of *n*. So, there is a possible world w_2 where Woody was made of *o* – a matter with sufficient overlap with *n* – and w_2 is possible with respect to w_1. Once again, the paradox arises by applying exactly the same method repeatedly. It will lead us to a possible world w_n where the table was made of a matter with 98% overlapping the matte used to make the table in w_{n-1}, but it is completely different from the matter that Woody was actually made. Although w_n is a possible world with respect to w_{n-1} – there was 98% overlapping between the matter from which the table was made in w_n with respect to w_{n-1}–, w_n is not possible with respect to the actual world. Just because the matter from what Woody was actually made from (*m*) and the matter which the table of w_n was

[11] Leslie, Sarah, «Essence, Plenitude and Paradox», *Philosophical Perpectives: Metaphysics* (2011), 277-296, p. 282.

made from overlaps *m* in a smaller portion of the what is required. Therefore, if there is some, it is a different table rather than Woody in w_n.

A simplified version of the non-vague paradox may help us to understand what is going on here and perhaps it makes clear that vagueness is not the source of the modal paradox currently discussed. For formulation of this new version of the non-vague paradox we must make a small variation in our second modal principle T_{F1} to get T_{F2} that is explicitly defined in the following way:

> [T_{F2}] A change smaller than 1/3 in the original matter of an artefact is tolerable.

Let us take in consideration a hypothetical chair and stipulate that it has (for simplifying) just three equally relevant parts: the back, the seat and the legs. Now we can regard that a chair in w_0 is made of the three specific parts: $back_1$, $seat_1$ and $legs_1$. By applying T_{F2}, a change in the chair of w_0 is allowed, if it is not bigger than 1/3. So, $back_1$, $seat_1$ and $legs_2$ is a possible way for the chair of w_0 be. In other words, there is a world w_1 possible to w_0 where the chair was made of the parts $back_1$, $seat_1$ and $legs_2$. One more time applying T_{F2}, the chair of w_1 may be made of the parts $back_1$, $seat_2$ and $legs_2$. This possible way for the chair be overlaps the chair of w_1 in 2/3 (which is exactly what is required by T_{F2}), which means that there exist a possible world w_2, possible relatively to w_1, where the chair was made of $back_1$, $seat_2$ and $legs_2$. Finally, the chair of w_2 could have been made of the three parts $back_2$, $seat_2$ and $legs_2$ (once again, applying T_{F2}). So, there is a possible world w_3 where the chair was made of $back_2$, $seat_2$ and $legs_2$. In spite of being a possible world relatively to w_2, given that the chair of w_3 overlaps the parts of the chair of w_2 in 2/3 (completely obeying the requirement of T_{F2}), w_3 itself is not a possible world relatively to w_0, given that the modification advanced over the chair of w_3 is total; which is not acceptable by T_{F2}. So, the chair made of $back_2$, $seat_2$ and $legs_2$ is an impossible way for the chair of w_0 to be, the chair of w_3 does not preserve any part of the original chair from w_0. Actually, even the chair of w_2 is not a possible way for the chair of w_0 to be, given that it does not preserve the minimum required by T_{F2} (the overlap between the chair of w_2 and the chair of w_0 is just 1/3 what is smaller than the portion required by $T_{F2.}$)

I hope that the last two versions of the paradox that have been just presented are capable to invite the reader to think carefully about the centrality of the notion of vagueness to the emergence of the modal paradox as it has been discussed in this paper. And (who knows) maybe the reader will arrive at the same conclusion that I got some time ago: even though I accept that some

kinds of theories available in the literature to deal with vagueness may give us insightful ideas to deal with modal paradox as well, the vagueness itself does not play a central role in the modal paradox emergence.

Technical features of the paradox

Although the main aim of this paper is to explore philosophical issues regarding the modal paradox, a more detailed and technical approach may be asked. This section is supposed to fill the gap and gives us the tools required to provide some solutions to the paradox. By "technical approach" I mean a formal derivation of an explicit paradox from the modal principals following explicit steps allowed by modal logical. Just by doing this, it will make clear how strong the (alethic) modal logic must be to paradox emergence.

There are infinitely many different ways to derive an explicit paradox from the modal principles E and T, in the following we will be faced with one of them. The demonstration of the paradox that I am going to present is composed of two phases that drive us from the set up (basically, the modal principles just mentioned) to a contradiction through almost uncontroversial steps. The first one, the tolerance phase, syntactically reflects the semantic idea of the relevant object slowly changing its material composition through a line of possible world in such a way that is allowed by T. The second one, the S4 phase, is the phase where the object of the far away world is showed to be possible by S4 modal principle, which says that if something is possibly possibly possible, then it is possibly possible and also that if something is possibly possible, then it is possible and so on. That is exactly the English expression of the syntactic notion performed in modal logic by the converse of the characteristic axiom schema of S4 $\ulcorner\Diamond\Diamond\Phi\rightarrow\Diamond\Phi\urcorner$ [12]. Which states that for any formula (including, obviously, formulas with prior modal operators) that is possibly possible is possible.[13]

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[12] Alternatively (and more often, actually), S4 is presented with its necessity axiom, namely, $\ulcorner\Box\Phi\rightarrow\Box\Box\Phi\urcorner$ which states if any formula is necessary, then it is necessarily necessary. In S4 necessity presentation, the converse axiom follows as a theorem from the system and the other way around happens as well. In a few words, S4 necessity axiomatization and S4 possibility axiomatization are equivalent modal logical systems, which means that their sets of theorems are just one and the same set. Furthermore, a version of the paradox may be derived from necessity axiomatization with or without its converse. For those more interested in logical proofs, the process of finding these proofs might be a funny time killer.

[13] It is worth noting that for any proof constructed in the weaker S4 modal logic system, there will be a S5 proof available as well, once all S4 theorems are S5 theorems. In other words, the set of S4 theorems are a subset of S5 theorems.

In the following derivation "M" is a predicate meaning "composed of" and is to be read as "M_ _" "_ is composed of _". The constant *a* is used to refer a specific object (a table, for instance) and *m*, similarly, is used as a name for a specific hunk of matter. Thus, "M*am*" should be read as "*a* is composed of *m*". Underwriting is used to get available an infinitely great stock of letter to refer to specifics hunk of matter and overwriting is indicating iteration of modal operators. And, finally, the logical connectors are the usual ones of modal logic.

1) Mam_0
2) $\neg \Diamond Mam_n$ An instance of [E]
3) $\Box (Mam_0 \rightarrow \Diamond Mam_1)$ An instance of [T][14]

} Setup

4) $\Box \Box (\Diamond Mam_1 \rightarrow \Diamond\Diamond Mam_2)$
5) $\Box^3 (\Diamond^2 Mam_2 \rightarrow \Diamond^3 Mam_3)$
6) $\Box^4 (\Diamond^3 Mam_3 \rightarrow \Diamond^4 Mam_4)$
n) $\Box^n (\Diamond^{n-1} Mam_{n-1} \rightarrow \Diamond^n Mam_n)$

} Tolerance phase

n+$S4_1$) $\Diamond^n Mam_n \rightarrow \Diamond^{n-1} Mam_n$
n+$S4_2$) $\Diamond^{n-1} Mam_n \rightarrow \Diamond^{n-2} Mam_n$
n+$S4_3$) $\Diamond^{n-2} Mam_n \rightarrow \Diamond^{n-3} Mam_n$
n+$S4_n$) $\Diamond Mam_n$

} S4 phase

n+$S4_{n+1}$) $\Diamond Mam_n \wedge \neg\Diamond Mam_n$

} Contradiction

As it has been said above, this derivation of the paradox is constructed in just two phase to lead us from the set up (which is basically a formal version of the T and E) to the the contradiction that says both is possible and impossible to a specific object *a* to be made of a hunk of matter m_n (matter of the step *n*). Although the step 3 is not properly an instance of T itself, it is an instance (in the object language) of a formal metalinguistic version of T which has the following form $\Box^\eta (\Diamond^{\eta-1} M\alpha\mu^{\eta-1} \rightarrow \Diamond^\eta M\alpha\mu_\eta)$ and we may call it "meta-

[14] Although it is not properly an instance of [T] itself, it is an instance (in the object language) of a formal metalinguistic version of [T] which has the following form $\Box^\eta (\Diamond^{\eta-1} M\alpha\mu^{\eta-1} \rightarrow \Diamond^\eta M\alpha\mu_\eta)$ and we call it "metalinguistic modal principle" or just (MMP) (the Greece letters are intentionally used here for both generalize the principle as much as possible and distinguish it from its instances in the object language). From the semantic point of view, it basically says that for any possible world in a chain and for any object (artefact) inhabiting them and any material composition of that objects, if we take in consideration a specific object made of a specific hunk of matter, then a small change in that matter is possible to that object. Which means that in next possible world of the chain that object is made of a slightly different new hunk of matter.

linguistic modal principle" (MMP). (Greece letters are intentionally used here for both generalize the principle as much as possible and distinguish it from its instances in the object language).[15] In the first phase, tolerance phase, is just applied MMP *n* times to reach in the step *n* the formula $\Box^{n} (\Diamond^{n-1}\mathrm{Mam}_{n-1} \rightarrow \Diamond^{n} \mathrm{Mam}_{n})$. In other words, in the tolerance phase the relevant object has its material composition slowly changed to reach a matter from which it could not have been made from, i.e., it is an impossible way to the relevant object to be. The next phase, S4 phase, is aimed to show that the impossible way reached in the step *n* is not actually impossible, though. And it follows quite straightforwardly from application of S4 modal logic because although it is really impossible to the object *a* to be made of m_n, it is possibly possible to *a* to be made of m_n. Thus, if it is possibly possible to *a* to be made of m_n, then, applying the converse of S4 modal logic *n* times, it is possible to *a* to be made of m_n. Which is exactly what is not allowed by modal principle E, as it is said by its instance in the step 2 of the derivation. Therefore, we reach in the step $n+S4_{n+1}$ an explicit contradiction, so the paradox is demonstrated.

Solutions

In this section I am going to advance some solutions to the modal paradox available in the literature. I am going to begin with some more general solutions which go against either essentialist theses or modal discourse itself. And then, I am going to present some of the more important solutions properly to the modal paradox.

Rejection to essentialism

One may argue that E and T (or any version of them) will lead us to paradox and we have to reject one or even both of them to avoid the paradox. Such a strategy is not impossible to be carried out, but it is not as easy as it seems to be at first sight. We can formulate two different answers for those who intend to carry on with that kind of strategy. The former is arguing that if you reject E

[15] From the semantic point of view, it basically says that for any possible world in a chain and for any object (artefact) inhabiting them and any material composition of that objects, if we take in consideration a specific object made of a specific hunk of matter, then a small change in that matter is possible to that object. Which means that in next possible world of the chain that object is made of a slightly different new hunk of matter.

(or a version of it), then you are in fact rejecting the essentialist thesis that the material composition of an artefact is essential to it. From my point of view, that is the same as rejecting the conclusion of an argument before the beginning of the discussion and cannot help to solve the paradox. Of course, if we stick to a philosophical thesis which leads us to a paradox and the paradox is impossible to be solved, then it is more rational to give up on the thesis and keep the consistency of both our theory and our set of beliefs. Nevertheless, that is completely different to just rejecting the essentialist thesis before anything else. Moreover, that kind of strategy will work out just in the case where the rejection of the essentialist thesis completely avoids the paradox and it is not obvious that this is the case here. From that direction arises the second answer.

Quine's global rejection of modal discourse

There remains one different strategy to (dis)solve the paradox: Quine's solution: global rejection of modal discourse. Quine did not give much attention to the paradox, but by taking what he says in a short paper (1976) about the subject, the case just considered by us motivated him (in addition to more general philosophical positions) to stick what we can call 'global rejection of modal discourse'. The global rejection of modal discourse is an extreme way to deal with the paradox. Basically, a quinean (and Quine by himself) does not need to give an answer to the paradox just because for him the modal discourse are globally mistaken. Our discourse about necessities, contingencies, possibilities, essences and anything else which is related to modality cannot be reconciled with our better explanation of the reality; the natural sciences. If our modal discourse cannot to be harmonized with the discourse of natural science, then we must give up on it. Of course, we can still be talking about essences and modalities in art, novels, our day-to-day dialogues and so on. But, we should banish all mentions of that kind of discourse of our explanation of the reality at the cost of introducing unintelligible terms in our explanation, which was supposed to be as clear and precise as possible. This way of dealing with the paradox is not obviously inconsistent or false, but it has a high price to pay. One that holds it will have to cut out a big part of our typical philosophical and ordinary discourse about modalities.

Salmon's solution

Salmon's solution is quite simple. Actually, it is just based on the rejection of the modal principle of the axiom schema of S4 modal system which holds that what is possibly possible is possible. Or its necessity counterpart axiom, what is necessary is necessarily necessary. In the Kripke's style semantics terms, Salmon's solution is based on the idea that the accessibility relation between possible world is not transitive, which means that in his framework $_{w0}R_{w1}$ and $_{w1}R_{w2}$ will not entail $_{w0}R_{w2}$. From Salmon's point of view, the fact that a sentence 'possibly possible φ' is true with respect to the actual world does not entail that the sentence 'possible φ' is also true with respect to the actual world. In other words, the fact that there are some possible worlds accessible relatively to some possible worlds accessible from the actual world where φ is true does not entail that those worlds where φ is true are themselves accessible relatively to the actual world. How can the restriction on modal logic systems settle the paradox? The reader may have already got the point here. The solution follows really straightforwardly from the restriction on the accessibility relation between possible worlds or, equivalently, from the rejection of modal principle asserted by axiom schema of S4. Looking at the demonstration given above, Salmon's solution just blocks all the phase S4 of the proof. Once S4 phase is not allowed no contradiction arises. Despite being quite simple, Salmon's solution rejects transitivity on accessibility relation between possible worlds which means that neither S4 nor S5 modal logic system are adequate modelling our reasoning about possibility and necessity. Maybe B, an alternative system to S4, or even T, a system weaker than B and S4, should be taken as adequate. One may think it is a too high price to pay.

Counterpart theoretical strategy

There remains at least one more strategy available: counterpart theoretical strategy. It has two completely different implementations in the work of Graeme Forbes and David Lewis. Unfortunately, all details and differences between their theories cannot be discussed here, but the essential shared idea might provide one more way out to the modal paradox for those who the previous solutions are not satisfactory.

The main idea of the counterpart theoretical framework is that identity trough possible worlds is substituted for a weaker notion of counterpart trough

possible worlds. While identity is a transitive relation between an object and itself in different possible worlds, counterpart is intentionally an intransitive relation between an object and its counterpart in different possible worlds. So, there is no identity in the counterpart theory, which means that one object does not hold identity with any other object in different possible worlds, instead it holds just the weaker relation of counterpart. Once counterpart relation is not transitive, it is easy to see how someone who stick with it can solve the paradox.

If relation between objects trough possible worlds is not transitive and a table made of a slightly different matter from that it was actually made is possible, then, one counterpart theorist might argue, the table has a counterpart in a different possible world w1 made exactly of that matter. And the table of w_1 has a counterpart in w_2 made of a slightly different from what it was made in w_1, and so on. However, there is nothing that allows us to suppose that the possible world w_2 is possible relatively to the actual possible world. Since the concept of counterpart is a substitute to identity and it is intentionally intransitive, modal paradox that shows up in normal modal logic does not seem to be a problem in the counterpart theoretical modal logic based on the weaker notion of counterpart.

Different implantations of counterpart theory will give us different way to deal with the paradox. One of them may block the S4 phase by not allowing the transitivity. Another one, may reject the tolerance phase arguing that once identity is not present, the idea of an object possibly made of a different matter is not properly understood. This very same idea can be used to avoid even the modal principle T which is present in the step 3 of the demonstration. In a more accurate modal principle bounded by counterpart theory T should not be permitted, once it says that one object is made of a different matter in a different possible world. One more accurate version of T in the counterpart theory should say that a counterpart of it is made of a different matter in a different possible world.

One may argue that counterpart theory cannot adequately capture our intuitive notion of possibility, once in counterpart theory we are not saying that is possible for that table to be made of a different matter, but there is a different table, a counterpart of that, made of a different matter. And, finally, one may not be prepared to pay to price to move from normal modal logic with identity to a completely different modal logic based on the concept of counterpart.

As it has been mentioned, this paper aimed to explore the conceptual field related to modal paradox and modality itself providing for the reader an effective background for further works in the subject matter. I hope this target has been achieved.

Bibliographic references

Chandler, Hugh (1976), «Plantinga and the Contingently Possible», *Analysis*, 106-109.
Chisholm, Roderick (1967), «Identity Through Possible Worlds: Some Questions», *Noûs*, 01-08.
___ (1973), «Parts as Essential to Their Wholes», *The Review of Metaphysics*, 581-603.
Forbes, Graeme (1984), «Two Solutions to Chisholm's Paradox», *Philosophical Studies*, 171-187.
___ (1986), *The Metaphysics of Modality,* Oxford University Press, Oxford.
Kripke, Saul (1980), *Naming and Necessity,* Blackwell, Oxford.
Leslie, Sarah (2011), «Essence, Plenitude and Paradox», *Philosophical Perpectives: Metaphysics*, 277-296.
Quine, W. V. (1976), «Worlds Away», *The Journal of Philosophy*, 859-863.
Salmon, Nathan (1986), «Modal Paradox: Parts and Counterparrts, Points and Counterpoints», *Midwest Sudies in Philosophy: Studies in Essencialism*, 75-120.
___ (1989), «The Logic of What Might Have Been», *Philosophical Review*, 3-34.
___ (1981), *Reference and Essence*, Prometheus Books, Amherst.
___ (1979), «How Not to Derive Essentialism from the Theory of Reference», *The Journal of Philosophy*, 703-725.